Fluids and FE

Morning (Fluid Mechanics)
A. Flow measurement
B. Fluid properties
C. Fluid statics
D. Energy, impulse, and momentum equations
E. Pipe and other internal flow

Afternoon (Depends on Discipline)
A. Bernoulli equation and mechanical energy balance
B. Hydrostatic pressure
C. Dimensionless numbers (e.g., Reynolds Number)
D. Laminar and turbulent flow
E. Velocity head
F. Friction losses (e.g., pipes, valves, fittings)
G. Pipe networks
H. Compressible and incompressible flow
I. Flow measurement (e.g., orifices, Venturi meters)
J. Pumps, turbines, and compressors
K. Non-Newtonian flow
L. Flow through packed beds

7% of FE Morning Session

Up to 15% of FE Afternoon Session
### FE Review - March 27, 2015
Steve Burian (Civil & Environmental Engineering)
General Exam - Fluids

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<td>Flow Measurement</td>
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<tr>
<td>Pumps, Turbines, and Compressors</td>
<td>Yes - Pump &amp; Turbines</td>
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</table>
1. If 6 m$^3$ of oil weighs 47 kN, calculate its specific weight, density, and specific gravity.

2. 10.0 L of an incompressible liquid exert a force of 20 N at the earth’s surface. What force would 2.3 L of this liquid exert on the surface of the moon? The gravitational acceleration on the surface of the moon is 1.67 m/s$^2$.

A. 0.39 N
B. 0.78 N
C. 3.4 N
D. 4.6 N
Fluids

- Fluids - substances in liquid or gas phase

- Fluids cannot support shear; they deform continuously to minimize applied shear forces
DENSITY, SPECIFIC VOLUME, SPECIFIC WEIGHT, AND SPECIFIC GRAVITY

The definitions of density, specific volume, specific weight, and specific gravity follow:

\[ \rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} \]

\[ \gamma = \lim_{\Delta V \to 0} \frac{\Delta W}{\Delta V} \]

\[ \gamma = \lim_{\Delta V \to 0} g \frac{\Delta m}{\Delta V} = \rho g \]

Also, \( SG = \gamma / \gamma_w = \rho / \rho_w \), where

- \( \rho \) = density (also called mass density),
- \( \Delta m \) = mass of infinitesimal volume,
- \( \Delta V \) = volume of infinitesimal object considered,
- \( \gamma \) = specific weight,
  = \( \rho g \),
- \( \Delta W \) = weight of an infinitesimal volume,
- \( SG \) = specific gravity,
- \( \rho_w \) = density of water at standard conditions
  = 1,000 kg/m\(^3\) (62.43 lbf/ft\(^3\)), and
- \( \gamma_w \) = specific weight of water at standard conditions,
  = 9,810 N/m\(^3\) (62.4 lbf/ft\(^3\)), and
  = 9,810 kg/(m\(^2\)\(\cdot\)s\(^2\)).
3. The viscosity of a fluid is

A. the dimensionless ratio of the weight of a body to the weight of an equal volume of a substance taken as a standard.
B. the weight of a unit volume of a substance.
C. caused by surface tension and depends on the relative magnitudes of the cohesion of a liquid and the adhesion of the liquid to the walls of the containing vessel.
D. that property which determines the amount of its resistance to a shearing force.
Viscosity

- **Shear stress** ($\tau$): force required to slide one unit area layer of a substance over another

- **Viscosity** ($\mu$): measure of a fluid’s resistance to flow when acted upon by an external force (i.e., ease with which a fluid pours)

- As a fluid moves a shear stress is developed in it; magnitude is dependent on viscosity of fluid
**Newtonian fluids:** strains are proportional to the applied shear stress

**Non-Newtonian fluids:** fluid shear stress can be computed using the power law

For a power law (non-Newtonian) fluid
\[ \tau = K (dv/dy)^n, \]
where
\[ K = \text{consistency index, and} \]
\[ n = \text{power law index.} \]
\[ n < 1 \equiv \text{pseudo plastic} \]
\[ n > 1 \equiv \text{dilatant} \]

The kinematic viscosity is the ratio of the absolute viscosity to mass density:
\[ \nu = \frac{\mu}{\rho} \]
### Properties of Water (SI Metric Units)

<table>
<thead>
<tr>
<th>Temperature °C</th>
<th>Specific Weight $\gamma$, kN/m$^3$</th>
<th>Density $\rho$, kg/m$^3$</th>
<th>Absolute Dynamic Viscosity $\mu$, Pa·s</th>
<th>Kinematic Viscosity $\nu$, m$^2$/s</th>
<th>Vapor Pressure $p_v$, kPa</th>
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4. A capillary tube 5 millimeters in diameter has its end submerged in mercury. The capillary depression is 1 millimeter. If the angle made by the mercury (s.g. 13.57) and the tube wall is 140°, the surface tension of the mercury is most nearly:

A. 0.22 N/m  
B. 2.22 N/m  
C. 4.44 N/m  
D. 26.00 N/m  
E. 49.59 N/m

The capillary rise \( h \) is approximated by

\[ h = \frac{4\sigma \cos \beta}{\gamma d}, \]

where

- \( h \) = the height of the liquid in the vertical tube,
- \( \sigma \) = the surface tension,
- \( \beta \) = the angle made by the liquid with the wetted tube wall,
- \( \gamma \) = specific weight of the liquid, and
- \( d \) = the diameter of the capillary tube.
Surface Tension

- “skin” that seems to form on free surface of a fluid; caused by intermolecular cohesive forces and is known as surface tension, $\sigma$
- Surface tension - tensile force between two points a unit distance apart on the surface

Surface tension $\sigma$ is the force per unit contact length

\[ \sigma = \frac{F}{L}, \text{ where} \]

- $\sigma$ = surface tension, force/length,
- $F$ = surface force at the interface, and
- $L$ = length of interface.
Capillarity

- **Capillary action**: caused by surface tension between liquid and a vertical solid surface.
- **Adhesive forces** between liquid molecules and surface > cohesive forces between liquid molecules; in water, adhesive forces cause fluid to attach itself to and *climb* solid vertical surface.

The capillary rise $h$ is approximated by

$$h = \frac{4\sigma \cos \beta}{\gamma d},$$

where

- $h$ = the height of the liquid in the vertical tube,
- $\sigma$ = the surface tension,
- $\beta$ = the angle made by the liquid with the wetted tube wall,
- $\gamma$ = specific weight of the liquid, and
- $d$ = the diameter of the capillary tube.
5. A device measuring the pressure in a closed vessel registers a vacuum of 310 millimeters of mercury (s.g. 13.57) when the absolute atmospheric pressure is 100 kPa. Using the notion that 760 millimeters of mercury is equivalent to 101.3 kPa, the absolute pressure in the vessel is most nearly:

A. 41.3 kPa
B. 58.8 kPa
C. 13.3 kPa
D. −41.3 kPa
E. −58.8 kPa
Pressure

- Hydrostatic pressure: pressure of fluid on immersed object or container walls
- Pressure = force per unit area of surface:

\[ P = \frac{F}{A} \]
Pressure

- **Gage pressure**: measured relative to a reference pressure - typically local atmospheric pressure
- **Absolute pressure**: measured relative to a perfect vacuum
- Absolute, gage, and atmospheric pressure are related as follows:

\[ P_{\text{abs}} = P_{\text{gage}} + P_{\text{atm}} \]
Pressure

Munson et al. (2002)

Fluid Properties
Fluid Statics
Fluid Dynamics
Energy, Friction Loss, and Pipe Flow
Momentum and Drag
6. The gage pressure at a point 3 m below the surface of an open topped, water filled tank is most nearly:

A. 681.25 kPa
B. 358.54 kPa
C. 187.20 kPa
D. 98.10 kPa
E. 29.43 kPa
Hydrostatic Pressure

\[ \Delta P = \gamma \Delta h \]

\( \Delta P \) = change in pressure
\( \gamma \) = specific weight of fluid
\( \Delta h \) = change in depth in fluid

***Incompressible fluid at rest***
7. The pressure in pipe A shown below is most nearly:

A. 6.3 kPa  
B. 5.8 kPa  
C. 4.2 kPa  
D. −5.8 kPa  
E. −6.3 kPa
Manometry

- Measure pressure or pressure differences
- **Differential manometers**: both ends connected to pressure sources
- **Open manometers**: one end open to the atmosphere

For a simple manometer,

\[ p_0 = p_2 + \gamma_2 h_2 - \gamma_1 h_1 = p_2 + g (\rho_2 h_2 - \rho_1 h_1) \]

If \( h_1 = h_2 = h \)

\[ p_0 = p_2 + (\gamma_2 - \gamma_1) h = p_2 + (\rho_2 - \rho_1)gh \]

Note that the difference between the two densities is used.
Another device that works on the same principle as the manometer is the simple barometer.

\[ p_{\text{atm}} = p_A = p_v + \gamma h = p_B + \gamma h = p_B + \rho gh \]

\[ p_v = \text{vapor pressure of the barometer fluid} \]
8. A bar of soap with the dimensions 10 cm long, 5 cm wide, and 3 cm tall is floating in a basin of water with 8 mm extending above the surface. If the water density at the present temperature is 997 kg/m³, the density of the soap is most nearly:

A. 1510 kg/m³  
B. 1359 kg/m³  
C. 731 kg/m³  
D. 135.9 kg/m³  
E. 73.1 kg/m³

9. In a static liquid, the difference in pressure between two different elevations is:

A. Equal to the difference in elevation multiplied by the fluid density.  
B. Equal to the difference in elevation multiplied by the specific weight of the fluid.  
C. Equal to the total depth of the fluid.  
D. The same in any direction.  
E. A function of time.
Buoyancy

Buoyant force = weight of fluid displaced and is directed vertically upward (Archimedes’ Principle):

\[ F_b = \gamma V_d \]

where
- \( F_b \) = buoyant force
- \( \gamma \) = specific weight of fluid
- \( V_d \) = displaced volume of fluid
Displaced Volume
Solving Buoyancy Problems

- If object at rest in fluid, then use equation of static equilibrium in vertical direction, $\Sigma F_y = 0$

- Buoyant force passes vertically through centroid of displaced volume; called the center of buoyancy.
10. The force on a one-meter wide section of a dam holding back 24 m of water is most nearly:

A. 17.97 kN  
B. 2,825 kN  
C. 3,532 kN  
D. 5,094 kN  
E. 5,660 kN
Fluid Forces on Surfaces

- Pressure on horizontal plane is uniform over surface
- Resultant force of pressure distribution acts through center of pressure of surface and is:

\[ R = PA \]

- \( R \) = resultant vertical force
- \( P \) = pressure on horizontal surface
- \( A \) = area of submerged horizontal surface
Fluid Forces on Surfaces

Forces on a submerged plane wall. (a) Submerged plane surface. (b) Pressure distribution.

The pressure on a point at a distance \( Z' \) below the surface is

\[ p = p_o + \gamma Z', \text{ for } Z' \geq 0 \]

If the tank were open to the atmosphere, the effects of \( p_o \) could be ignored.
Fluid Forces on Surfaces

Free Surface

\[ R = P_{\text{avg}} A = \gamma h_c A \]

\[ y_R = \frac{I_{xc}}{Ay_c} + y_c \]

\[ x_R = \frac{I_{xyc}}{Ay_c} + x_c \]
11. Laminar flow exists in a pipe. We know that

A. The Reynolds Number is less than 2000.
B. The velocity profile is linear.
C. The shear stress distribution is linear.
D. The pipe is smooth.
Laminar and Turbulent Flow

**Laminar Flow:**
- Relatively low velocities
- No mixing or a very small degree of mixing
- Fluid appears to flow in continuous layers with no interaction between the layers

**Turbulent Flow:**
- Relatively high velocities
- High degree of mixing
- Fluid motion appears chaotic
Flow Distribution

The velocity distribution for \textit{laminar flow} in circular tubes or between planes is

$$v(r) = v_{\text{max}} \left[ 1 - \left( \frac{r}{R} \right)^2 \right],$$

where

- $r$ = the distance (m) from the centerline,
- $R$ = the radius (m) of the tube or half the distance between the parallel planes,
- $v$ = the local velocity (m/s) at $r$, and
- $v_{\text{max}}$ = the velocity (m/s) at the centerline of the duct.

- $v_{\text{max}} = 1.18 \overline{v}$, for fully turbulent flow
- $v_{\text{max}} = 2 \overline{v}$, for circular tubes in laminar flow and
- $v_{\text{max}} = 1.5 \overline{v}$, for parallel planes in laminar flow, where
- $\overline{v}$ = the average velocity (m/s) in the duct.
Reynolds Number

**REYNOLDS NUMBER**

\[ Re = \frac{vD\rho}{\mu} = \frac{vD}{\nu} \]

\[ Re' = \frac{\sqrt{(2 - n)D^n \rho}}{K \left( \frac{3n + 1}{4n} \right)^n 8^{(n - 1)}} \]

- \( \rho \) = the mass density,
- \( D \) = the diameter of the pipe, dimension of the fluid streamline, or characteristic length.
- \( \mu \) = the dynamic viscosity,
- \( \nu \) = the kinematic viscosity,
- \( Re \) = the Reynolds number (Newtonian fluid),
- \( Re' \) = the Reynolds number (Power law fluid), and
- \( K \) and \( n \) are defined in the Stress, Pressure, and Viscosity section.

**Flow in Noncircular Conduits**

Analysis of flow in conduits having a noncircular cross section uses the *hydraulic radius* \( R_H \), or the *hydraulic diameter* \( D_H \), as follows

\[ R_H = \frac{\text{cross-sectional area}}{\text{wetted perimeter}} = \frac{D_H}{4} \]
Reynolds Number

Circular Pipe Flow

Re < 2000 laminar flow
2000 < Re < 4000 transition region
Re > 4000 turbulent flow

Open Channel

Re < 500 laminar flow
500 < Re < 2000 transition region
Re > 2000 turbulent flow

Flow through a pipe is generally characterized as laminar for Re < 2,100 and fully turbulent for Re > 10,000, and transitional flow for 2,100 < Re < 10,000.
12. The hydraulic radius of a non-circular conduit is defined as

A. The radius of an equivalent circular conduit.
B. The ratio of the cross-sectional area of flow to the wetted perimeter.
C. The ratio of the Reynolds Number to the roughness coefficient.
D. The hydraulic diameter of an equivalent circular conduit.
E. The ratio of the conduit area to its perimeter.

13. If the speed of an incompressible fluid is 4 m/s in a 240-mm-diameter pipe that enters a 160-mm-diameter pipe, what will the speed in the 160-mm-diameter pipe most nearly be?

A. 16.0 m/s
B. 10.7 m/s
C. 9.00 m/s
D. 4.00 m/s
E. 2.00 m/s
14. Water flows through a multi-sectional pipe placed horizontally on the ground. The velocity is 3.0 m/s at the entrance and 2.1 m/s at the exit. What is the pressure difference between these two points? Neglect friction.

A. 0.2 kPa  
B. 110 kPa  
C. 28 kPa  
D. 2.3 kPa

15. Select the false statement for the Bernoulli equation.

A. It is valid for unsteady flow.  
B. It is valid along a streamline.  
C. It is valid in an inertial coordinate system.  
D. It is valid for inviscid flow.
One-Dimensional Flows

The Continuity Equation
So long as the flow $Q$ is continuous, the continuity equation, as applied to one-dimensional flows, states that the flow passing two points (1 and 2) in a stream is equal at each point,

$$A_1v_1 = A_2v_2.$$

$$Q = Av$$

$$\dot{m} = \rho Q = \rho Av,$$ where

$Q$ = volumetric flow rate,

$\dot{m}$ = mass flow rate,

$A$ = cross section of area of flow,

$v$ = average flow velocity, and

$\rho$ = the fluid density.

For steady, one-dimensional flow, $\dot{m}$ is a constant. If, in addition, the density is constant, then $Q$ is constant.
The Field Equation is derived when the energy equation is applied to one-dimensional flows. Assuming no friction losses and that no pump or turbine exists between sections 1 and 2 in the system,

\[
\frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 = \frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 \quad \text{or}
\]

\[
\frac{P_2}{\rho} + \frac{v_2^2}{2} + z_2 g = \frac{P_1}{\rho} + \frac{v_1^2}{2} + z_1 g, \text{ where}
\]

- \( P_1, P_2 = \) pressure at sections 1 and 2,
- \( v_1, v_2 = \) average velocity of the fluid at the sections,
- \( z_1, z_2 = \) the vertical distance from a datum to the sections (the potential energy),
- \( \gamma = \) the specific weight of the fluid (\( \rho g \)), and
- \( g = \) the acceleration of gravity.
16. If an incompressible flow is carried in a pipe where the elevation and cross-sectional area remains constant between two points a certain distance apart, the pressure drop in the pipe between the two points is considered due to:

A. Atmospheric pressure  
B. Fluid density  
C. Velocity head change  
D. Friction  
E. Gravity
17. The Darcy-Weisbach friction factor \((f)\) is a function of:

A. The kinematic viscosity, velocity, and the Reynolds number.

B. The flow rate, dynamic viscosity, and the roughness factor.

C. The cross-sectional area and the wetted perimeter.

D. The Reynolds number, the roughness factor, and pipe diameter.

E. The average velocity, pipe length, pipe diameter, and gravity.
18. Water flows through a 10-cm-dia, 100-m-long pipe connecting two reservoirs with an elevation difference of 40 m. The average velocity is 6m/s. Neglecting minor losses, the friction factor is

A. 0.020
B. 0.022
C. 0.024
D. 0.026
Mechanical Energy Equation

The energy equation for incompressible flow is

\[
\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g} + h_f \quad \text{or}
\]

\[
\frac{p_1}{\rho g} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{v_2^2}{2g} + h_f
\]

\[
h_f = \text{the head loss, considered a friction effect, and all remaining terms are defined above.}
\]

If the cross-sectional area and the elevation of the pipe are the same at both sections (1 and 2), then \( z_1 = z_2 \) and \( v_1 = v_2 \). The pressure drop \( p_1 - p_2 \) is given by the following:

\[
p_1 - p_2 = \gamma h_f = \rho gh_f
\]
The Darcy-Weisbach equation is
\[ h_f = f \frac{L}{D} \frac{v^2}{2g}, \] where

- \( f = f(Re, e/D) \), the Moody or Darcy friction factor,
- \( D \) = diameter of the pipe,
- \( L \) = length over which the pressure drop occurs,
- \( e \) = roughness factor for the pipe, and all other symbols are defined as before.

Valid for laminar and turbulent flow

A chart that gives \( f \) versus \( Re \) for various values of \( e/D \), known as a Moody or Stanton diagram, is available at the end of this section.
Moody Chart

<table>
<thead>
<tr>
<th>Material</th>
<th>e (ft)</th>
<th>e (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Riveted steel</td>
<td>0.003–0.03</td>
<td>0.9–9.0</td>
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<tr>
<td>Concrete</td>
<td>0.001–0.01</td>
<td>0.3–3.0</td>
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<tr>
<td>Cast iron</td>
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<td>Galvanized iron</td>
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<td>Commercial steel or wrought iron</td>
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<td>0.046</td>
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<tr>
<td>Drawn tubing</td>
<td>0.00005</td>
<td>0.0015</td>
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</table>
Minor Loss

Head losses also occur as the fluid flows through pipe fittings (i.e., elbows, valves, couplings, etc.) and sudden pipe contractions and expansions.

\[
\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g} + h_f + h_{f, \text{fitting}}
\]

\[
\frac{p_1}{\rho g} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{v_2^2}{2g} + h_f + h_{f, \text{fitting}}, \text{ where}
\]

\[h_{f, \text{fitting}} = C \frac{v^2}{2g}, \text{ and } \frac{v^2}{2g} = 1 \text{ velocity head}
\]

Specific fittings have characteristic values of \(C\), which will be provided in the problem statement. A generally accepted nominal value for head loss in well-streamlined gradual contractions is

\[h_{f, \text{fitting}} = 0.04 \frac{v^2}{2g}\]
17. The Darcy-Weisbach friction factor (f) is a function of:

\[ f = \frac{64}{Re} \]

where \( f \) is the friction factor, \( Re \) is the Reynolds number, and \( \frac{Dv}{\mu} \) is the hydraulic diameter divided by the fluid's viscosity.
19. The locus of elevations that water will rise in a series of pitot tubes is called

A. The hydraulic grade line.
B. The energy grade line.
C. The velocity head.
D. The elevation head.
HYDRAULIC GRADIENT (GRADE LINE)
The hydraulic gradient (grade line) is defined as an imaginary line above a pipe so that the vertical distance from the pipe axis to the line represents the pressure head at that point. If a row of piezometers were placed at intervals along the pipe, the grade line would join the water levels in the piezometer water columns.

ENERGY LINE (BERNOULLI EQUATION)
The Bernoulli equation states that the sum of the pressure, velocity, and elevation heads is constant. The energy line is this sum or the “total head line” above a horizontal datum. The difference between the hydraulic grade line and the energy line is the $v^2/2g$ term.
HGL and EGL

Total Head or Energy Grade Line (EGL)

Velocity Head \( \left( \frac{v^2}{2g} \right) \)

Pressure Head \( \left( \frac{P}{\gamma} \right) \)

Elevation Head \( (z) \)

Z = 0
Topic: Pumps and Turbines

Problems 20 and 21: A 2-m diameter, 200-m long, cast-iron pipe transports water from a reservoir with surface elevation 726 m to an 89% efficient turbine which has its outlet at 696 m. The turbine operates such that the flow rate is 6 $m^3/s$. Use kinematic viscosity $\nu = 10^{-6} \ m^2/s$.

20. Approximate the losses up to the inlet of the turbine.

A. 20 m
B. 10 m
C. 2.5 m
D. 0.25 m

21. What is the expected power output of the turbine?

A. 1550 kW
B. 960 kW
C. 270 kW
D. 43.2 kW
Pump-Turbines

Net head added to system by mechanical device

\[
\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} + h_s - h_L = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g}
\]

**PUMP POWER EQUATION**

\[
\dot{W} = Q \frac{\gamma h}{\eta} = Q \rho gh / \eta, \text{ where}
\]

- \(Q\) = volumetric flow (m\(^3\)/s or cfs),
- \(h\) = head (m or ft) the fluid has to be lifted,
- \(\eta\) = efficiency, and
- \(\dot{W}\) = power (watts or ft-lbf/sec).
22. A 15-m-wide, 1.2-m deep river feeds a reservoir from above (Q = 6 m$^3$/s). Estimate the river’s slope if the Manning $n$ is 0.035.

A. 0.013  
B. 0.107  
C. 0.0013  
D. 0.00107
OPEN-CHANNEL FLOW AND/OR PIPE FLOW

Manning’s Equation

\[ v = \left( \frac{k}{n} \right) R^{2/3} S^{1/2}, \text{ where} \]

\[ k = 1 \text{ for SI units,} \]

\[ k = 1.486 \text{ for USCS units,} \]

\[ v = \text{velocity (m/s, ft/sec),} \]

\[ n = \text{roughness coefficient,} \]

\[ R_H = \text{hydraulic radius (m, ft), and} \]

\[ S = \text{slope of energy grade line (m/m, ft/ft).} \]
23. The force exerted by a 25-mm-diameter stream of water against a flat plate held normal to the stream’s axis is 645 N. What is the flow?

A. 0.0178 m³/s
B. 0.0056 m³/s
C. 0.7183 m³/s
D. 0.0141 m³/s
THE IMPULSE-MOMENTUM PRINCIPLE
The resultant force in a given direction acting on the fluid equals the rate of change of momentum of the fluid.

\[ \Sigma F = Q_2 \rho_2 v_2 - Q_1 \rho_1 v_1, \text{ where} \]

- \( \Sigma F \) = the resultant of all external forces acting on the control volume,
- \( Q_1 \rho_1 v_1 \) = the rate of momentum of the fluid flow entering the control volume in the same direction of the force, and
- \( Q_2 \rho_2 v_2 \) = the rate of momentum of the fluid flow leaving the control volume in the same direction of the force.
Impulse-Momentum

\[ \sum F_x = Q_2 \rho_2 V_{2x} - Q_1 \rho_1 V_{1x} \]

\[ \sum F_y = Q_2 \rho_2 V_{2y} - Q_1 \rho_1 V_{1y} \]

\[ \sum F_z = Q_2 \rho_2 V_{2z} - Q_1 \rho_1 V_{1z} \]

Sum of the external forces

Net rate of momentum entering control volume
24. The branched pipeline shown below has a flow in pipe A of 10 cubic feet per second. The following table gives the characteristics of each of the pipes.

![Diagram of pipeline with branches A and B]

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>16 in.</td>
<td>24 in.</td>
</tr>
<tr>
<td>Length</td>
<td>2,000 ft</td>
<td>3,000 ft</td>
</tr>
<tr>
<td>F</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>Area</td>
<td>1.3963 ft²</td>
<td>3.142 ft²</td>
</tr>
</tbody>
</table>

The flow in pipe B is most nearly:

A. 8.77 cfs
B. 21.60 cfs
C. 27.56 cfs
D. 77.94 cfs
E. 81.30 cfs
The same head loss occurs in each branch as in the combination of the two. The following equations may be solved simultaneously for \( v_A \) and \( v_B \):

\[
h_L = f_A \frac{L_A}{D_A} \frac{v_A^2}{2g} = f_B \frac{L_B}{D_B} \frac{v_B^2}{2g}
\]

\[
(\pi D^2/4) v = (\pi D_A^2/4) v_A + (\pi D_B^2/4) v_B
\]

The flow \( Q \) can be divided into \( Q_A \) and \( Q_B \) when the pipe characteristics are known.
<table>
<thead>
<tr>
<th>Topic</th>
<th>Reviewed</th>
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<tbody>
<tr>
<td><strong>Morning Session</strong></td>
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<tr>
<td>Fluid Properties</td>
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<tr>
<td>Fluid Statics</td>
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<tr>
<td>Energy, Impulse, Momentum</td>
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<tr>
<td>Pipe and Other Internal Flow</td>
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<td>Flow Measurement</td>
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<tr>
<td><strong>Afternoon Session</strong></td>
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<tr>
<td>Bernoulli Equation</td>
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<td>Mechanical Energy Balance</td>
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<tr>
<td>Hydrostatic Pressure</td>
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<tr>
<td>Dimensionless Numbers</td>
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<tr>
<td>Laminar and Turbulent Flow</td>
<td>Yes</td>
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<tr>
<td>Veolocity Head</td>
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<tr>
<td>Friction Losses (Pipes, Valves, Etc.)</td>
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<td>Pipe Networks</td>
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<tr>
<td>Compressible Flow</td>
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<tr>
<td>Incompressible Flow</td>
<td>Yes</td>
</tr>
<tr>
<td>Flow Measurement</td>
<td>No</td>
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<td>Pumps, Turbines, and Compressors</td>
<td>Yes - Pump &amp; Turbines</td>
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<td>Non-Newtonian Flow</td>
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<td>Flow Through Packed Beds</td>
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<tr>
<td>Lift and Drag</td>
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<tr>
<td>Airfoil Theory</td>
<td>No</td>
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</table>
Good Luck!!!

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