Homework 2: Total Probability, Independence, and Bayes’ Rule

Instructions: Submit a single R Markdown file (.Rmd) of your work on Canvas by 11:59pm on the due date. You may also submit diagrams, drawings, etc. as image files (.png, .jpg, .gif)—they must be formatted into your .Rmd document (we won’t look at them separately). Be sure to show all the work involved in deriving your answers! If you just give a final answer without explanation, you may not receive credit for that question.

You may discuss the concepts with your classmates, but write up the answers entirely on your own. Do not look at another student’s answers, and do not show your answers to anyone.

1. (15 pts) Exercise 3.5 from the book.

2. (15 pts) Simulate the above problem (Exercise 3.5) 10,000 times using R code. What is the estimated probability from your simulation that both balls are red?

3. (25 pts) A meteorologist has a prediction model for whether a hurricane will make landfall. When a hurricane does actually make landfall, the model correctly predicts this 90% of the time. When a hurricane does not make landfall, the model incorrectly predicts that it will hit land 15% of the time. The unconditional probability that a hurricane will hit land is 70%.

   a. Draw a probability tree for this problem. *Hint:* the first-level branches should be the hurricane makes landfall or not, the second-level branches should be the meteorologist predicts landfall or not. **Show all probabilities, including the multiplication rules at the end:** you should have 10 probabilities in your tree.

   b. What is the total probability that the meteorologist will predict landfall of a hurricane?

   c. If the meteorologist predicts that a certain hurricane will hit land, what is the probability that it actually will?

4. (20 pts) Independence: **Show the formula you use to check these!**

   a. You roll a six-sided die. Are the following two events independent?
   
   \[ A = \text{“the number is less than or equal to 4”}, \]
   \[ B = \text{“the number is odd”}. \]

   b. You flip two coins. Are the following two events independent?
   
   \[ A = \text{“you flipped repeats (HH or TT)”}, \]
   \[ B = \text{“you flipped at least one heads”}. \]

5. (25 pts) A doctor sees a patient who may have a particular heart disease. According to the patient’s family history and other risk factors, the doctor decides there is a 30% chance that the patient has the disease. The doctor then takes a blood test that turns out positive for the disease. However, there is a 10% chance that the test is positive when a patient does not have the disease. When a patient does have the disease, there is an 85% chance that the test will be positive. What is the probability that the patient has the disease given the positive test? *Hint:* Think about what the two events are, and use Bayes’ Rule.
Optional Extra Credit (15 pts) Using R code, simulate the “Boy Born on Tuesday” problem with 1,000,000 simulations. For each simulation, you will need to randomly draw 2 sexes (boy or girl) and draw 2 days of the week (Sun - Sat). Now, if you restrict yourself to only the simulations that include a boy born on Tuesday (event $B$ from class), what is the probability that both children are boys (event $A$ from class)? In other words, what is your simulated probability of $P(A \mid B)$?