More Practice

1. The bus line that you take in to school claims that it is on average only 5 minutes late. Based on how many times you have been late to class, you think the bus is actually later than that. So you record the bus arrival times over 9 days. On average you find the bus is $\bar{x}_9 = 6$ minutes late, with a sample standard deviation of $s_9 = 1.5$ minutes. You now want to perform a hypothesis test on this data, to see if the bus really is later than their claim.

(a) What is the null hypothesis, $H_0$, and the alternate hypothesis, $H_1$?

(b) What type of statistic would you compute to test this hypothesis? What is the value of it? (Hint: it’s a simple number)
(c) Say you choose a significance level of $\alpha = 0.05$. Below is a graph of the pdf for the sample statistic in part (b). Label it with the following information:

i. The critical value for this test comes out to either $-2.26$ or $+2.26$. Pick the correct one, and mark it on the $x$-axis of the graph.

ii. Draw on the graph how the $p$-value would be computed from your test statistic in part (b). (Hint: I’m looking for you to shade an area of the graph.)

(d) Would you reject the null hypothesis? (Just answer yes or no.)

(e) Now, instead of a hypothesis test, compute a 99% confidence interval of the average. Let $F$ denote the cdf for the appropriate Student’s $t$ distribution. You will need one of the following values:

$$F(0.99) = 0.826 \quad F(0.995) = 0.827 \quad F^{-1}(0.99) = 2.82 \quad F^{-1}(0.995) = 3.25$$

**Hint:** Your confidence interval should be symmetric about the sample mean, and you don’t need to do the arithmetic to simplify the final answer.
2. Say you are given a random sample, \( Z_1, Z_2, \ldots, Z_n \), where each random variable is defined as \( Z_i = \frac{1}{2}X_i^2 + \frac{1}{2}Y_i^2 \), with both \( X_i \sim N(\mu, 1) \) and \( Y_i \sim N(\mu, 1) \).

(a) What is the expectation \( E[X_i^2] \)? **Hint:** Use the formula for variance of a random variable, and the fact that you know \( E[X_i] \) and \( \text{Var}(X_i) \) because \( X_i \sim N(\mu, 1) \).

(b) What is \( E[Z_i] \)? **Hint:** Use part (a), and the fact that \( E[X_i^2] = E[Y_i^2] \).

(c) Say you want to estimate \( \mu^2 \) with the mean statistic: \( \hat{\mu}^2 = Z_n \). What is the bias of this statistic? **Hint:** Use part (b), you should get a simple number.