Review of Engineering Thermodynamics

Universal Balance Equation for Any Extensive Property

Accumulation = transport + generation

Integrated form for some period of time:

\[
\begin{align*}
\text{final amount} - \text{initial amount} &= \text{amount entering} - \text{amount leaving} + \text{amount generated} - \text{amount consumed} \\
\end{align*}
\]

Rate form:

\[
\begin{align*}
\text{rate of change} &= \text{rate of transport in} - \text{rate of transport out} + \text{rate of generation} - \text{rate of consumption} \\
\end{align*}
\]

Mass Balance

- Unsteady balance for CV

\[
\frac{dm_{CV}}{dt} = \sum m_i - \sum m_e \\
\Delta m_{CV} = m_2 - m_1 = \sum m_i - \sum m_e
\]

- Steady balance for CV

\[
0 = \sum m_i - \sum m_e \\
0 = \sum m_1 - \sum m_2
\]

- Balance for closed system

\[
\frac{dm_{sys}}{dt} = 0 \\
\Delta m_{sys} = m_2 - m_1 = 0
\]

- Averaged flow

\[
m = \rho_{av} \cdot Vel_{av} \cdot A = \frac{Vel_{av} \cdot A}{v_{av}} = \frac{V}{v_{av}}
\]

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Energy Balance

\[ e = u + \frac{1}{2} V e l^2 + g z, \quad h = u + P v \]

- **Unsteady balance for CV**

\[
\frac{dE_{CV}}{dt} = \dot{Q}_{in,net} + \dot{W}_{in,net} + \sum_{inlets} \dot{m}_i \left( h + \frac{V e l^2}{2} + g z \right)_i - \sum_{exits} \dot{m}_e \left( h + \frac{V e l^2}{2} + g z \right)_e
\]

- **Steady balance for CV**

\[
0 = \dot{Q}_{in,net} + \dot{W}_{in,net} + \sum_{inlets} \dot{m}_i \left( h + \frac{V e l^2}{2} + g z \right)_i - \sum_{exits} \dot{m}_e \left( h + \frac{V e l^2}{2} + g z \right)_e
\]

- **Balance for closed system**

\[
\frac{dE_{sys}}{dt} = \dot{Q}_{in,net} + \dot{W}_{in,net} \quad \Delta E_{sys} = Q_{in,net} + W_{in,net}
\]

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Entropy Balance

There is only one form of entropy.

- **Unsteady balance for CV**

\[
\frac{dS_{CV}}{dt} = \sum_{j=1}^n \frac{\dot{Q}_{in,j}}{T_j} + \sum_{inlets} \dot{m}_i s_i - \sum_{exits} \dot{m}_e s_e + \dot{S}_{gen}
\]

\[ \dot{S}_{gen} > 0 \text{ irreversible process} \]

\[ \dot{S}_{gen} = 0 \text{ reversible process} \]

\[ \dot{S}_{gen} < 0 \text{ impossible process} \]

- **Steady balance for CV**

\[
0 = \sum_{j=1}^n \frac{\dot{Q}_{in,j}}{T_j} + \sum_{inlets} \dot{m}_i s_i - \sum_{exits} \dot{m}_e s_e + \dot{S}_{gen}
\]

- **Balance for closed system**

\[
\frac{dS_{sys}}{dt} = \sum_{j=1}^n \frac{\dot{Q}_{in,j}}{T_j} + \dot{S}_{gen}
\]

\[ \Delta S_{sys} = m(s_2 - s_1) = \int \frac{\dot{Q}_{in}}{T} + \dot{S}_{gen} \]

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### Combined Entropy and Energy Balance

\[ du = \delta q + \delta w \quad ds = \frac{\delta q}{T} + ds_{gen} \]

\[ du = \delta q_{rev} + \delta w_{rev} \quad ds = \frac{\delta q_{rev}}{T} \quad \delta w_{rev} = -Pdv \]

\[ Tds = du + Pdv \quad \text{or} \quad ds = \frac{du}{T} + \frac{Pdv}{T} \]

An alternate form follows from the relation

\[ d(Pv) = Pdv + vdp \]

\[ Tds = dh - vdp \quad \text{or} \quad ds = \frac{dh}{T} - \frac{vdp}{T} \]

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### Combined Entropy and Energy Balance for Ideal Gases

\[ \Delta s = \int_{T_1}^{T_2} c_v(T) \frac{dT}{T} + R \ln \frac{V_2}{V_1} \quad \Delta s = \int_{T_1}^{T_2} c_p(T) \frac{dT}{T} - R \ln \frac{P_2}{P_1} \]

For constant or averaged heat capacities,

\[ \Delta s = c_{v,av} \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1} \quad \Delta s = c_{p,av} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \]

where \( T_{av} = \frac{T_1 + T_2}{2} \) and \( c_{av} = c(T_{av}) \)

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Equations for Work

Reversible boundary work, closed system:

\[ w_{rev, in} = -\int_{V_1}^{V_2} P \, dv \]

Steady-flow, reversible work, open system:

\[ w_{rev, in} = \frac{W}{m} = \int_{V_1}^{V_2} v \, dP + \frac{V c_l^2 - V c_l^2}{2} + g(z_2 - z_1) \]

Models of Working Substances

Solid/Liquid
- \( C_p = C_v = C \)
- \( \Delta u = \Delta h = C \Delta T \)

Gases
- \( P V = RT \)
- \( P \) and \( T \) are absolute.
- \( C_p = \text{constant} \)
- \( \Delta u = C_v \Delta T \)
- \( \Delta h = C_p \Delta T \)

Phase-change fluids (water & refrigerants)
- \( x = \frac{m_{sup}}{m_{mol}} = \frac{v - v_f}{v_g - v_f} \)

Mixtures (advanced)

Perfect gas
- \( P V = RT \)
- \( P \) and \( T \) are absolute.
- \( C_p = \text{constant} \)
- \( \Delta u = C_v \Delta T \)
- \( \Delta h = C_p \Delta T \)

Ideal gas
- \( P V = RT \)
- \( P \) and \( T \) are absolute.
- \( C_p = f(T) \)
- \( C_v = C_p - R \)
- \( \Delta u = \int_{T_i}^{T_f} C_v \, dT \)
- \( \Delta h = \int_{V_i}^{V_f} C_p \, dT \)

Real gas
- \( Z = f(T_r, P_r) = \frac{V_{\text{actual}}}{V_{\text{ideal}}} \)
- \( Z = \frac{P_{\text{actual}}}{RT} \)
- \( T_r = \frac{T}{T_i} \)
- \( P_r = \frac{P}{P_i} \)

P and \( T \) are absolute.


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