FE Statics Review

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MCE room 2016 (through 2000B door)
Position and Unit Vectors

If you wanted to express a 100 lb force that was in the direction from A to B in vector form, then you would find $e_{AB}$ and multiply it by 100 lb.

$$r_{AB} = 7i - 7j - k \text{ (units of length)}$$

$$e_{AB} = (7i - 7j - k)/\sqrt{99} \text{ (unitless)}$$

Then:

$$F = F\ e_{AB}$$

$$F = 100 \text{ lb } \{(7i - 7j - k)/\sqrt{99}\}$$
Another Way to Define a Unit Vectors

Direction Cosines

The $\theta$ values are the angles from the coordinate axes to the vector $\mathbf{F}$. The cosines of these $\theta$ values are the coefficients of the unit vector in the direction of $\mathbf{F}$.

If $\cos(\theta_x) = 0.500$, $\cos(\theta_y) = 0.643$, and $\cos(\theta_z) = -0.580$, then:

$$\mathbf{F} = \mathbf{F} (0.500\mathbf{i} + 0.643\mathbf{j} - 0.580\mathbf{k})$$
Trigonometry

• Sin θ = opposite/hypotenuse
• Cos θ = adjacent/hypotenuse
• Tan θ = opposite/adjacent
Trigonometry Continued

\[ \sum \text{angles} = a + b + c = 180^\circ \]

Law of sines: \( \frac{\sin a}{A} = \frac{\sin b}{B} = \frac{\sin c}{C} \)

Law of cosines: \( C^2 = A^2 + B^2 - 2AB\cos c \)
Products of Vectors – Dot Product

\[ \overrightarrow{U} \cdot \overrightarrow{V} = UV \cos(\theta) \quad \text{for } 0 \leq \theta \leq 180^\circ \]

( This is a scalar.)

In Cartesian coordinates:

\[ \overrightarrow{U} \cdot \overrightarrow{V} = U_x V_x + U_y V_y + U_z V_z \]

In Cartesian coordinates to find \( \theta \)?

\[ UV \cos(\theta) = U_x V_x + U_y V_y + U_z V_z \]

\[ \theta = \cos^{-1} \left[ \frac{(U_x V_x + U_y V_y + U_z V_z)}{UV} \right] \]
Products of Vectors – Dot Product

\[ UV \cos(\theta) = U_x V_x + U_y V_y + U_z V_z \]
\[ \theta = \cos^{-1} \left[ \frac{(U_x V_x + U_y V_y + U_z V_z)}{UV} \right] \]

However, usually we use vectors so that we **do not** have to deal with the angle between the vectors.
Products of Vectors – Dot Product

\[ \vec{U} \cdot \vec{V} = UV \cos(\theta) \quad \text{for } 0 \leq \theta \leq 180^\circ \]

How could you find the projection of vector \( \vec{U} \), in the direction of vector \( \vec{V} \)?

The direction of vector \( \vec{V} \)?

\[ \vec{e}_V = \frac{\vec{V}}{V} \rightarrow e_V = 1 \]

\[ \vec{U} \cdot \vec{e}_V = (U)(1)\cos(\theta) = U\cos(\theta) = \text{the answer} \]

The projection of \( \vec{U} \) in the direction of \( \vec{V} \) is \( \vec{U} \) dotted with the unit vector in the direction of \( \vec{V} \).
Products of Vectors – Cross Product

Also called the vector product

\[
\mathbf{V} \times \mathbf{V} = \mathbf{U} \mathbf{V} \sin(\theta) \mathbf{e}
\]

for \(0 \leq \theta \leq 180^\circ\)

Note: \(\mathbf{U} \times \mathbf{V} = -\mathbf{V} \times \mathbf{U}\)

If \(\theta = 0\) \(\mathbf{U} \times \mathbf{V} = 0\)

If \(\theta = 90^\circ\) \(\mathbf{U} \times \mathbf{V} = \mathbf{U} \mathbf{V} \mathbf{e}\)

In Cartesian coordinates:

\[
\mathbf{i} \times \mathbf{i} = 0 \quad \mathbf{i} \times \mathbf{j} = \mathbf{k} \quad \mathbf{j} \times \mathbf{i} = -\mathbf{k}
\]

etc.
Products of Vectors – Cross Product

If \( \mathbf{U} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k} \) and \( \mathbf{V} = -\mathbf{i} + 2\mathbf{j} - \mathbf{k} \), then find \( \mathbf{U} \times \mathbf{V} \).

\[
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
2 & 3 & -1 \\
-1 & 2 & -1 \\
\end{vmatrix}
\]

Multiply and use opposite same sign.

\[
3\mathbf{k} \quad 2\mathbf{i} \quad 2\mathbf{j} \quad -3\mathbf{i} \quad \mathbf{j} \quad 4\mathbf{k}
\]

Then add all six values together, but note that \( \mathbf{i} \) can add to \( \mathbf{i} \), \( \mathbf{j} \) to \( \mathbf{j} \), and \( \mathbf{k} \) to \( \mathbf{k} \), but \( \mathbf{i} \) cannot add to \( \mathbf{j} \), etc.

\[
\mathbf{U} \times \mathbf{V} = -\mathbf{i} + 3\mathbf{j} + 7\mathbf{k} \quad \text{units}
\]
Products of Vectors
Mixed Triple Product

\[ \mathbf{U} \times \mathbf{V} \cdot \mathbf{W} \]

\[ \mathbf{U} \times \mathbf{V} = \mathbf{U} \mathbf{V} \sin(\theta) \mathbf{e} \]

Projection of \( \mathbf{U} \mathbf{V} \sin \theta \) on the direction of \( \mathbf{W} \).
Products of Vectors
Mixed Triple Product

Note: No $i \ j \ k$

Otherwise, same procedure

$$U \times V \cdot W = \begin{vmatrix}
U_x & U_y & U_z \\
V_x & V_y & V_z \\
W_x & W_y & W_z \\
\end{vmatrix}$$
Pulley Problem
(fric
tionless pins)

\[ \Sigma F_y = 0 = 2T - T_1 - mg \]
\[ T_1 = 2T - mg \]

\[ \Sigma F_y = 0 = 2T_1 - mg - m_A g \]
\[ = 2(2T - mg) - mg - m_A g \]

\[ T = (3m + m_A)g/4 \]
Moment of a Force

2-dimensional, in the plane of the screen

The moment of $\mathbf{F}$ with respect to point $O$ is the perpendicular distance from point $O$ to the line of action of $\mathbf{F}$ times the magnitude $|\mathbf{F}|$ of $\mathbf{F}$, where positive is counter-clockwise (CCW).
Moment of a Force

Then:

\[ \mathbf{r} \times \mathbf{F} = rF \sin(\theta) \mathbf{e} \]

\[ = r \sin(\theta) F \mathbf{e} \]

\[ = (\mathbf{D})(\mathbf{F}) \mathbf{e} = \mathbf{M} \]

Note that since \( \mathbf{M} = DF \rightarrow D = \mathbf{M}/F \)
The magnitude of the force $\mathbf{F}$ is 100N. Determine the moment of $\mathbf{F}$ about point O and about the x-axis.

$\mathbf{M}_O = \mathbf{r}_{OB} \times \mathbf{F}$

You should determine: $\mathbf{M}_O = (-20.9\mathbf{i} + 25.1\mathbf{k}) \text{ N} \cdot \text{m}$  ?? j term??

Then find $\mathbf{M}_O \times \text{axis}$

$\mathbf{M}_O \times \text{axis} = -20.9 \text{ N} \cdot \text{m}$  ?? vector form??
Couples

**F** causes translation and, in general, rotation.

Let \(-F\) be:

- Equal in magnitude to \(F\)
- Opposite direction of \(F\)
- Not collinear with \(F\).

Then \(F\) and \(-F\) form a plane, and cause rotation, but no translation.

Pick an arbitrary point in 3-D space and draw position vectors from that point to a point on the line of action of \(F\) (point A), and a point on the line of action of \(-F\) (point B). Draw position vector \(r_{BA}\).

Then: \[ r_{0B} + r_{BA} = r_{0A} \]

Or: \[ r_{BA} = r_{0A} - r_{0B} \]

\[ M_0 = (r_{0A} \times F) + (r_{0B} \times (-F)) = (r_{0A} - r_{0B}) \times F \]

\[ M_0 = r_{BA} \times F \]

The forces \(F\) and \(-F\) form a **couple**. The moment of a couple is the same about every point in space. Therefore, the moment of a couple is a **free vector**. Also, two couples that have the same moment are **equivalent**.
Equivalent Systems

1. \( \mathbf{F} \)

2. \(-\mathbf{F}\)

3. \(\mathbf{F} \), equivalent to 1

4. \(\mathbf{F} \), equivalent to 1
Review

For static equilibrium: $\Sigma F = 0$ and $\Sigma M = 0$

In 3-D you have 6 independent equations:
- 3 force and 3 moment

in 2-D you have 3 independent equations:
- 2 force and 1 moment
  - or 1 force and 2 moment (often)
  - or 0 force and 3 moment (often)
Reactions (2-D)

Rough surface or pinned support

Smooth surface or roller support

Fixed, built-in, or cantilever support

Rope or cable (tension)

Spring (tension or compression)
\[ \Sigma F_x = 0 = A_x - 10\cos(80) \]
\[ A_x = 1.74 \text{ N} \]
\[ \Sigma F_y = 0 = A_y - 10\sin(80) \]
\[ A_y = 9.85 \text{ N} \]
\[ \Sigma M_A = 0 = M_A - 10\sin(80)(4) \]
\[ M_A = 39.4 \text{ N} \cdot \text{m} \]
3-D Beam Solution – Given $T$ find $M_A$

Is this bar properly constrained and statically determinate?

Yes and yes

Options:
- Sum forces in a direction
- Sum moments about a point
- Sum moments about an axis

Which option is best here?

$$\Sigma M_A = 0 = M_{Ax}i + M_{Ay}j + M_{Az}k + (r_{AB} \times T)$$

Then group the $i$, $j$, and $k$ components and solve.
3-D Beam Solution – Given $W$, find $T$

Is this bar properly constrained and statically determinate?

Yes (?) and no

Options:
- Sum forces in a direction
- Sum moments about a point
- Sum moments about an axis

Which option is best here?
- Sum moments about the axis.
Special Case ➔ Two-force Bodies

If a body in equilibrium has forces at two and only two locations and no moments, then the forces are equal in magnitude, opposite in direction, and along the line of action between the two points.
Special Case ➔ Three-force Bodies

If a body in equilibrium has forces at three and only three locations and no moments, then the forces are either concurrent or parallel. Usually, you can use the trigonometric or graphical techniques of Chapter 2.
Trusses

The truss model requires that all members be two-force members.

This further requires the assumptions that:

- the connections between the members are frictionless pins.
- all forces and reactions are applied at connection points.

As a result, the forces in prismatic members are along the members.
Work truss problems efficiently.

First look at the physics of the problem to see:

- if you can solve for the forces in any members by inspection.
- if you need to find the reactions.
- if there is symmetry in loading and geometry that can be used.

If the problem is not solved directly from the physics, then,

- use the method of joints to solve for the unknowns if they are near a known force that can be used in the solution.
- use the method of sections to solve for the unknowns if they are not near a known force that can be used in the solution.
What are the forces in members BC, DE, and AB?

By special case rules:
- BC = 0  answer
- DE = 100 lb (C)  answer

\[ \Sigma F_x = 0 = F_{AX} \]

By symmetry:

\[ F_{Ay} = \frac{[100 + (200)(2) + 1700]}{2} = 1100 \text{ lb} \]

\[ \Sigma F_y = 0 = 1100 \text{ lb} - AB \sin \left(45^\circ\right) \]

Therefore:
- AB = 1556 lb (C)  answer
What is the force in member BE? Note that I have provided the values of the reactions.

\[ \Sigma F_y^{+\uparrow} = 0 = 1100 \text{ lb} - \text{BE} \sin (45^\circ) \]

\[ \text{BE} = 1556 \text{ lb (T)} \]
Frames and machines

\[ \Sigma M_A = 0 = -(4.5)(700) + (7)(E_y) \]
\[ E_y = 450 \text{ N} \]
\[ \Sigma F_y = 0 = A_y - 700 + 450 \]
\[ A_y = 250 \text{ N} \]
\[ \Sigma F_x = 0 = A_x + E_x \quad \text{Can’t solve yet.} \]
Frames

\[ \Sigma M_C = 0 = - (4.5)(700) + (7)(E_y) \]
\[ E_y = 450 \text{ N} \]
\[ \Sigma F_y = 0 = A_y - 700 + 450 \]
\[ A_y = 250 \text{ N} \]
\[ \Sigma F_x = 0 = A_x + E_x \quad \text{Can’t solve yet.} \]
\[ A_x = 150 \text{ N} \]
Centroids

Integration in $dx$ and $dy$, which always can be done:

Note that $dA = dx \ dy$

$$\bar{x} = \frac{\int_A x \, dA}{\int_A dA} \quad \bar{y} = \frac{\int_A y \, dA}{\int_A dA}$$

$$A = \int_A dA = \int_0^b \int_0^h \, dx \, dy = \int_0^h \left[ x \right]_0^b \, dy = \int_0^h (b - 0) \, dy = \int_0^h b \, dy = b \int_0^h dy = b \ y|_0^h = b (h - 0) = bh$$

$$\int_A x \, dA = \int_0^h \int_0^b x \, dx \, dy = \int_0^h \frac{1}{2} x^2 \, dy = \frac{1}{2} \int_0^h (b^2 - 0) dy = \frac{1}{2} b^2 h$$

Similarly: \( \int_A y \, dA = \frac{1}{2} bh^2 \)

Therefore: \[ x = \frac{1}{2} \frac{b^2 h}{bh} = \frac{1}{2} b \quad y = \frac{1}{2} \frac{bh^2}{bh} = \frac{1}{2} h \]
What is the x-component of the centroid of the figure shown below, which is formed by the rectangle and triangle?

\[ \bar{x} = \frac{\sum(xA)}{\sum A} \]

<table>
<thead>
<tr>
<th>Figure</th>
<th>( \bar{x} ) (ft)</th>
<th>A (ft(^2))</th>
<th>( \bar{x}A ) (ft(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>3</td>
<td>7</td>
<td>21</td>
</tr>
<tr>
<td>Triangle</td>
<td>6</td>
<td>10.5</td>
<td>63</td>
</tr>
<tr>
<td>Sums</td>
<td></td>
<td>17.5</td>
<td>84</td>
</tr>
</tbody>
</table>

\[ \bar{x} = \frac{84}{17.5} = 4.8 \text{ feet} \]
Area Moments of Inertia

Moment of inertia about the x axis:
\[ I_x = \int y^2 \, dA \]  
Note y not x in the integral.

Moment of inertia about the y axis:
\[ I_y = \int x^2 \, dA \]

Polar moment of inertia:
\[ J_0 = \int r^2 \, dA \]
\[ = \int (x^2 + y^2) \, dA \]
\[ = I_x + I_y \]

Product of Inertia:
\[ I_{xy} = \int xy \, dA \]

Units are \text{length}^4
Radius of Gyration

The radius of gyration (k) is the distance at which the entire area would need to be located to give the same moment of inertia as the actual area.

\[ I_x = \int_A y^2 \, dA = \int_A k_x^2 \, dA = k_x \int_A dA = k_x^2 A \]

Therefore, \( k_x = \sqrt{\frac{I_x}{A}} \) Units are length.

Used in buckling computations, and found in tables of standard material shapes.
Find the moment of inertia of a rectangle about its base.

Moment of inertia about the x axis:

\[
I_x = \int_A y^2 dA = \int_0^h \int_a^{a+b} y^2 dx dy = \int_0^h y^2[(a+b)-a] dy \\
= b[h^3 - 0]/3 = 1/3(bh^3)
\]
Parallel axis theorem

The moment of inertia of a figure about any axis is equal to the moment of inertia of the figure about the parallel axis through the centroid of the figure plus the area times the distance between the axes squared.

\[ I_L = I_x' + Ad^2 \]

You **MUST** move to and from the centroidal axis only!
Find the moment of inertia of a rectangle about its centroidal axis $x'$. 

We found the moment of inertia about the $x$ axis:

$$I_x = \frac{1}{3}(bh^3)$$

But, $I_x = I_{x'} + Ad^2 \Rightarrow I_{x'} = I_x - Ad^2$

Therefore $I_{x'} = \frac{1}{3}(bh^3) - bh(h/2)^2 = \frac{1}{12}(bh^3)$
Composite Structures
Find the moment of inertia and radius of gyration of the figure about its base.

• Select the parts to use.
• Find the moment of inertia of each part about the base. I labeled the base as the x axis.
• Add the moments of inertia to get the total, which is the moment of inertia about the base.
• Find the radius of gyration by dividing the moment of inertia by the area, and taking the square root.
Composite Structures
Find the moment of inertia and radius of gyration of the figure about its base.

A way: Subtract the red rectangle from the blue.

\[ I_x = \frac{1}{3} (bh^3) = \frac{1}{3} [(6')(10'')^3] = 2000 \text{ in}^4 \]
\[ I_x = \frac{1}{12} (bh^3) + Ad^2 \]
\[ = \frac{1}{12} [(4')(6'')^3] + (4'')(6'')(7'')^2 = 1248 \text{ in}^4 \]
\[ I_{\text{xtotal}} = 2000 \text{ in}^4 - 1248 \text{ in}^4 = 752 \text{ in}^4 \]

\[ A = (6')(10'') - (4')(6'') = 36 \text{ in}^2 \]

\[ k_x = \left[\frac{(752 \text{ in}^4 )}{(36 \text{ in}^2)}\right]^{1/2} = 4.57 \text{ in} \]
The homogeneous slender bar has mass \( m \), cross-section area \( A \), and length \( l \). Use integration to find the mass moment of inertia of the bar about the axis \( L \) through its centroid.

\[
m = \rho Al
\]
\[
dm = \rho Adx \quad -l/2 \leq x \leq l/2
\]

\[
r^2 = x^2 \sin^2 \theta
\]

Integration from \(-l/2\) to \(l/2\) yields:

\[
I_L = \frac{1}{12} \rho A \sin^2 \theta l^3
\]

Then substitute \( m \) for \( \rho Al \) yielding:

\[
I = \frac{1}{12} \sin^2 \theta ml^2
\]
Static (3 Eq.)

Impending Motion (3 Eq. + $f_{\text{max}} = \mu_s N$)

Dynamic (only $f = \mu_k N$ unless constant velocity $\Rightarrow$ (3 Eq. + $f = \mu_k N$)
Type Problems

1. You don’t know where you are on the curve.
   Assume static equilibrium.
   Solve equilibrium equations to find $f$.
   Solve $f_{\text{max}} = \mu_s N$.
   If $f \leq f_{\text{max}}$, then the assumption is good.

2. Impending motion, and you know where.
   You have 4 equations, so just solve it.

3. Impending motion, but you don’t know where.
   Ask $\rightarrow$ what can happen?
Type Problems (continued)

4. $f > f_{\text{max}}$ Dynamics problem, but $f = \mu_k N$.

5. Dynamic, but with constant velocity $\rightarrow$ You can solve it just like a static equilibrium problem, and you may use $f = \mu_k N$.

6. Various things could happen.
   Ask $\rightarrow$ What could happen?
   Look at the physics of the problem.
   Develop a plan

What is the limiting case?
Given \( W_A \), \( W_B \), \( \theta \), and the coefficients of static friction between all surfaces, what is the largest force \( F \) for which the boxes will not slip?

If you are given a mass, then don’t forget to convert it to a weight.
“Belt friction” The box weighs 40 lb. The rope is wrapped 2 ¼ turns around the fixed wooden post. The coefficients of friction between the rope and post are $\mu_s = 0.1$ and $\mu_k = 0.08$.

(a) What minimum force is needed to support the stationary box?

(b) What force must be exerted to raise the box at a constant rate?

$$T_2 = T_1 e^{\mu_s \beta}$$

where: $T_2 > T_1$ You must decide this. $\mu$ could be either $\mu_s$ or $\mu_k$ $\beta$ is in radians, and often $> 2\pi$

Note also that, $\mu_s \beta = \ln(T_2/T_1)$

For (a) what are $T_1 = F$ For (b) 40 lb $T_2 = 40 \text{ lb}$ $\mu = 0.1$ 0.08 $\beta = 4.5\pi$ 4.5$\pi$
Water Pressure

Ignoring atmospheric pressure and the weight of the dam, what are the reactions at A and B

Width of dam (into figure) 10 ft
Weight density of water 62.4 lb/ft³
Water Pressure

Ignoring atmospheric pressure and the weight of the dam, what are the reactions at A and B

\[ F_w = (1)(10)(1.5)(62.4) \]
\[ = 936 \text{ lb} \]
\[ F_P = (1/2)[(3)(62.4)](10)(3) \]
\[ = 2808 \text{ lb} \]
\[ \Sigma M_B = 0 = -3A + (0.5)(936) + (1)(2808) \]
\[ A = 1092 \text{ lb} \]
\[ \Sigma F_x = 0 = 1092 - 2808 + B_x \]
\[ B_x = 1716 \text{ lb} \]
\[ \Sigma f_y = 0 = -936 + B_y \]
\[ B_y = 936 \text{ lb} \]

Sign convention?
SHEAR FORCE AND BENDING MOMENT DIAGRAM RULES
AND SHORT CUTS FOR BEAM LOADING

POSITIVE SIGN CONVENTION:
Distance along beam (x) plus to the right
Force (P) plus up
Couple (C) plus if counter-clockwise (CCW)
Distributed load (\( \omega \)) plus down
Shear (V) plus if rotate clockwise (CW) (up on left and down on right)
Moment (M) plus if up like aircraft wings in flight

Off the ends of the beam:
\[ V = 0 \text{ & } M = 0 \]

Cross a point load:
\[ V_{\text{+ (right)}} = V_{\text{- (left)}} + P \]
M continuous

Cross a point couple:
\[ V \text{ is not affected} \]
\[ M_{\text{+ (right)}} = M_{\text{- (left)}} - C \]

Through a distributed load:
(This includes \( \omega = 0 \))
\[ \Delta V = -\omega \text{ diagram area} \]
\[ \frac{dV}{dx} = -\omega \]
\[ \Delta M = V \text{ diagram area} \]
\[ \frac{dM}{dx} = V \]

Superposition applies if elastic.
Beams

We will discuss an example of a beam with point forces, a point couple, and a constant distributed load.

We will look at segments of the problem, but you need to think of it as a continuous solution with the figures aligned and the text and figures progressing down the page in an organized manner. The total solution is shown to the right.
Draw the shear force and bending moment diagrams.

To find the reactions, you may model the distributed load as a point load.

\[ \Sigma M_A = 0 = 10 - (10)(5) + E_y (8) \]
\[ E_y = 5 \text{ N} \]

\[ \Sigma f_y = A_y - 10 + 5 \]
\[ A_y = 5 \text{ N} \]

However, you must go back to the distributed load while completing the problem. To do otherwise would cause errors that might be unsafe.
Draw the shear force and bending moment diagrams.

To find the reactions, you may model the distributed load as a point load.

\[ \Sigma M_A = 0 = 10 - (10)(5) + E_y \]  
\[ E_y = 5 \text{ N} \]

\[ \Sigma f_y = A_y - 10 + 5 \]
\[ A_y = 5 \text{ N} \]

For \( 0 < x < 2 \text{m} \):  
Signs by beam convention

\[ V_x = 5 \text{ N} \]

\[ \Sigma M_x = 0 = -5x + M_x \]
\[ M_x = 5x \text{ (N\cdot m)} \]
Draw the shear force and bending moment diagrams.

For $0 < x < 2m$:
\[
V_x = 5 \text{ N} \\
M_x = 5x \text{ (N·m)}
\]

For $2m < x < 4m$:
\[
V_x = 5 \text{ N} \\
\Sigma M_x^+ = 0 = -5x + 10 + M_x \\
M_x = 5x - 10 \text{ (N·m)}
\]

For $4m < x < 6m$:
Find $V_x$ & $M_x$
Draw the shear force and bending moment diagrams.

For $0 < x < 2m$:
\[
V_x = 5 \text{ N} \\
M_x = 5x \text{ (N·m)}
\]

For $2m < x < 4m$:
\[
V_x = 5 \text{ N} \\
\Sigma M_x^+ = 0 = -5x + 10 + M_x \\
M_x = 5x - 10 \text{ (N·m)}
\]

For $4m < x < 6m$:
\[
\text{Find } V_x \text{ & } M_x \\
\Sigma f_y = 0 = 5 - (x - 4)(5) - V_x \\
V_x = 25 - 5x \text{ (N)} \\
\Sigma M_x^+ = 0 = -5x + 10 + M_x \\
\quad + (x - 4)(5)(x - 4)/2 \\
\text{force} = \text{area} \times \text{distance} \\
M_x = -(5/2)x^2 + 25x - 50 \text{ (N·m)}
\]

Note that the loading is not the same as it would be, if the point load through the centroid, that was used to find the reactions, were used here.
For \( 6 < x < 8 \):

\[
\Sigma f_y^+ = 0 = 5 - 10 - V_x
\]

\[
V_x = -5 \text{ N}
\]

\[
\Sigma M_x^+ = 0 = -5x + 10 + 10(x - 5) + M_x
\]

\[
M_x = -5x + 40 \quad \text{(N\cdot m)}
\]

When you are past the distributed load, then you can model it as a point load through the centroid.
For $0 < x < 2m$:
\[ V_x = 5 \text{ N} \]
\[ M_x = 5x \text{ (N}\cdot\text{m}) \]

For $2m < x < 4m$:
\[ V_x = 5 \text{ N} \]
\[ M_x = 5x - 10 \text{ (N}\cdot\text{m}) \]

For $4m < x < 6m$:
\[ V_x = 25 - 5x \text{ (N)} \]
\[ M_x = -\frac{5}{2} x^2 + 25x - 50 \text{ (N}\cdot\text{m}) \]

For $6 < x < 8$:
\[ V_x = -5 \text{ N} \]
\[ M_x = -5x + 40 \text{ (N}\cdot\text{m}) \]

Note that all sections of the plots are Constant or linear $f(x)$ except $4 < x < 6$. For it you can pick values.

For $x = 4m$ \[ M_x = 10 \text{ N}\cdot\text{m} \]
For $x = 5m$ \[ M_x = 12.5 \text{ N}\cdot\text{m} \]
For $x = 6m$ \[ M_x = 10 \text{ N}\cdot\text{m} \]